AIMING OF ARTILLERY AT STATIONARY AND MOVING TARGETS

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CONTENT

CHAPTER 1 MOTION OF ARROWS UNDER ACTION OF GRAVITY AND AIR RESISTANCE

1.1 INTRODUCTION

In this analysis, the motion of bodies with tail fins under the action of gravity and air resistance is considered. The action of gravity is represented by a force of magnitude W (kg) acting vertically downward, where W is the arrow weight. The air resistance is taken as a force opposite to the direction of motion and proportional to the square of the arrow velocity. Hence, the expression for air resistance may be written as kv^2 , where v (m/s) is the arrow velocity and k (kg-s^2 /m^2) is a drag constant.

The components of the forces and the accelerations are taken in the direction of the tangent and of the normal to the trajectory. Angle θ is taken arbitrarily to be the angle between the tangent to the trajectory and the x - axis (see Fig 1a). The component of the acceleration in the tangential direction is $\frac{dv}{dt}$ and in the normal direction is $\frac{v^2}{R}$ where R is the radius of curvature of the trajectory. Note that $\frac{1}{R} = \frac{d\theta}{ds}$ hence,

$$\frac{v^2}{R} = v^2 \frac{d\theta}{ds} = v^2 \frac{d\theta}{dt}\frac{dt}{ds}$$

Since v = ds/dt, then $\frac{v^2}{R} = v\frac{d\theta}{dt}$.

Now the equations of motion are obtained by equating the product of the mass and the acceleration to the force in both the tangential and normal directions.

1.2 ARROW ASCENT FROM ORIGIN TO APEX

From Fig 1 the following equations of motion may be written

$$m\frac{dv}{dt} = mg\sin\theta + kv^2 \qquad (1)$$

$$mv\frac{d\theta}{dt} = mg\cos\theta$$

The second equation may be re-written as

$$\frac{d\theta}{dt} = \frac{g}{v}\cos\theta \qquad (2)$$

Note that arrow mass m = W/g

Since $\frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt}$ equation 1 may be re-written as

$$m\frac{dv}{d\theta}\frac{g}{v}\cos\theta = mg\sin\theta + kv^2$$

or,

$$\frac{dv}{d\theta}\cos\theta - v\sin\theta = \frac{k}{W}v^3 \qquad (3)$$

Taking the horizontal component of the velocity $v_x = v\cos\theta$ as the unknown variable, then equation 3 may be re-written as

$$\frac{dv_x}{d\theta} = \frac{k}{W}\frac{v_x{}^3}{\cos^3\theta} \qquad (4)$$

or,

$$\frac{dv_x}{v_x{}^3} = \frac{k}{W}\frac{d\theta}{\cos^3\theta} \qquad (5)$$

Note that angle θ vary from α at the origin, to θ elsewhere. Now integrating equation 5 gives

$$\frac{1}{2v_x{}^2} = -\frac{k}{W}\int\frac{d\theta}{\cos^3\theta} + C$$

$$= -\frac{k}{W}\left[\frac{1}{2}\frac{\sin\theta}{\cos^2\theta} + \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right] + C \qquad (6)$$

At origin $\theta = \alpha$ and $v_x = v_i\cos\alpha$, where v_i is the initial velocity. Therefore,

$$C = \frac{1}{2v_i{}^2\cos^2\alpha} + \frac{k}{W}\left[\frac{1}{2}\frac{\sin\alpha}{\cos^2\alpha} + \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)\right]$$

Therefore,

$$\frac{1}{2v_x{}^2} = \frac{1}{2v_i{}^2\cos^2\alpha} + \frac{k}{W}\left[\frac{1}{2}\frac{\sin\alpha}{\cos^2\alpha} + \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)\right] - \frac{k}{W}\left[\frac{1}{2}\frac{\sin\theta}{\cos^2\theta} + \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right]$$

or,

$$\frac{1}{v_x{}^2} = \frac{1}{v_i{}^2\cos^2\alpha} + \frac{k}{W}\left[\left\{\frac{\sin\alpha}{\cos^2\alpha} + \log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)\right\} - \left\{\frac{\sin\theta}{\cos^2\theta} + \log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right\}\right]$$

Hence,

$$v_x = \frac{v_i \cos\alpha}{\sqrt{1 + \frac{kv_i^2 \cos^2 \alpha}{W}\left[\left\{\frac{\sin\alpha}{\cos^2\alpha} + \log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)\right\} - \left\{\frac{\sin\theta}{\cos^2\theta} + \log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right\}\right]}} \tag{7}$$

or, in parametric form

$$\begin{aligned} v_x &= v_i \cos\alpha \, f(\theta) \\ v_y &= v_i \cos\alpha \, f(\theta) \tan\theta \end{aligned} \tag{8}$$

At apex $\theta = 0$ and $v_x = v_0$ hence,

$$v_0 = \frac{v_i \cos\alpha}{\sqrt{1 + \frac{kv_i^2 \cos^2 \alpha}{W}\left[\frac{\sin\alpha}{\cos^2\alpha} + \log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)\right]}} \tag{9}$$

From equation 2, it follows that

$$dt = \frac{v}{g}\frac{d\theta}{\cos\theta} = \frac{v_x d\theta}{g\cos^2\theta} \tag{10}$$

Then substituting v_x of equation 8 in equation 10 and integrating, we get

$$t = \frac{v_i \cos\alpha}{g}\int_\alpha^\theta \frac{f(\theta)d\theta}{\cos^2\theta} \tag{11}$$

$$x = \int_0^t v_x dt = \frac{v_i^2 \cos^2\alpha}{g}\int_\alpha^\theta \frac{\{f(\theta)\}^2 d\theta}{\cos^2\theta} \tag{12}$$

$$y = \int_0^t v_y dt = \frac{v_i^2 \cos^2\alpha}{g}\int_\alpha^\theta \frac{\{f(\theta)\}^2 \sin\theta d\theta}{\cos^3\theta} \tag{13}$$

For constant k the problem of ascent is completely solved by equations 11, 12 and 13. Numerical evaluation of the above integrals is carried out by a computer program by the author.

1.3 ARROW DESCENT FROM APEX TO TARGET

The following two equations of motion may be written

$$m\frac{dv}{dt} = mg\sin\theta - kv^2 \tag{14}$$

$$mv\frac{d\theta}{dt} = mg\cos\theta$$

The term kv^2 has a minus sign because it acts in a direction opposite to $mg\sin\theta$.

From the second equation of equation 14, it follows

$$\frac{d\theta}{dt} = \frac{g}{v}\cos\theta \tag{15}$$

Since $\frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt}$, then the first equation of equation 14 may be re-written as

$$m\frac{dv}{d\theta}\frac{g}{v}\cos\theta = mg\sin\theta - kv^2$$

or,

$$\frac{dv}{d\theta}\cos\theta - v\sin\theta = -\frac{k}{W}v^3 \tag{16}$$

Taking the horizontal component of the velocity $v_x = v\cos\theta$ as the unknown variable, then equation 16 may be re-written as

$$\frac{dv_x}{d\theta} = -\frac{k}{W}\frac{{v_x}^3}{\cos^3\theta} \tag{17}$$

or,

$$\frac{dv_x}{{v_x}^3} = -\frac{k}{W}\frac{d\theta}{\cos^3\theta} \tag{18}$$

Now integrating equation 18 gives

$$\frac{1}{2{v_x}^2} = \frac{k}{W}\int\frac{d\theta}{\cos^3\theta} + C$$
$$= \frac{k}{W}\left[\frac{1}{2}\frac{\sin\theta}{\cos^2\theta} + \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right] + C$$

At apex $\theta = 0$ and $v_x = v_0$, hence,

$$C = \frac{1}{2{v_0}^2}$$

and,

$$\frac{1}{v_x^{\ 2}} = \frac{1}{v_0^{\ 2}} + \frac{k}{W}\left(\log\frac{1+\sin\theta}{\cos\theta} + \frac{\sin\theta}{\cos^2\theta}\right) \tag{19}$$

Therefore,

$$v_x = v_0 f(\theta) = \frac{v_0}{\sqrt{1 + \frac{kv_0^{\ 2}}{W}\left(\log\frac{1+\sin\theta}{\cos\theta} + \frac{\sin\theta}{\cos^2\theta}\right)}} \tag{20}$$

Since $v_y = v_x \tan\theta$, then,

$$\begin{aligned} v_x &= v_0\, f(\theta) \\ v_y &= v_0\, f(\theta)\tan\theta \end{aligned} \tag{21}$$

From equation 15 it follows that

$$dt = \frac{v}{g}\frac{d\theta}{\cos\theta} = \frac{v_x d\theta}{g\cos^2\theta} \tag{22}$$

Then substituting v_x in equation 20, and integrating equation 22, we get

$$t = \frac{v_0}{g}\int_0^\theta \frac{f(\theta)d\theta}{\cos^2\theta} \tag{23}$$

$$x = \int_0^t v_x dt = \frac{v_0^{\ 2}}{g}\int_0^\theta \frac{\{f(\theta)\}^2 d\theta}{\cos^2\theta} \tag{24}$$

$$y = \int_0^t v_y dt = \frac{v_0^{\ 2}}{g}\int_0^\theta \frac{\{f(\theta)\}^2 \sin\theta d\theta}{\cos^3\theta} \tag{25}$$

For constant k the problem of descent is completely solved by equations 23, 24 and 25. These integrals are numerically evaluated Simpson's rule is implemented in a computer program by the author for numerical evaluation of the integrals.

CHAPTER 2 EQUATIONS OF MOTION OF BODIES UNDER THE ACTION OF GRAVITY AND AIR RESISTANCE

2.1 INTRODUCTION

In this analysis, projectiles are assumed to be bodies moving under constant gravitational force and air resistance proportional to the square of their velocity. The velocity at any time t may be decomposed into vertical and horizontal components, and air resistance for each component is dealt with independently.

2.2 BODIES FALLING UNDER THE ACTION OF GRAVITY AND AIR RESISTANCE

A body of weight W falls under gravity against a resistance proportional to the square of its velocity. Its velocity v and displacement y at time t are determined as follows [1]

Let the resistance be kv^2 when the velocity is v. Then, if y is measured positive downwards,

$$\frac{W}{g}\frac{d^2y}{dt^2} = W - k_y v^2 \tag{1}$$

Writing
$$\frac{d^2y}{dt^2} = \frac{dv}{dt} \tag{2}$$

then

$$\int \frac{dv}{W - k_y v^2} = \int \frac{g}{W}dt + C \tag{3}$$

Hence

$$\frac{1}{2\sqrt{k_y W}}\log\left(\frac{\sqrt{\frac{W}{k_y}} + v}{\sqrt{\frac{W}{k_y}} - v}\right) = \frac{g}{W}t + C \tag{4}$$

Since v=0 at t=0,

$$C = \frac{1}{2\sqrt{Wk_y}}\log 1 = 0 \tag{5}$$

Therefore

$$\log\left(\frac{\sqrt{\frac{W}{k_y}} + v}{\sqrt{\frac{W}{k_y}} - v}\right) = 2g\sqrt{\frac{k_y}{W}}t \tag{6}$$

Whence

$$v = \sqrt{\frac{W}{k_y}}\left\{\frac{e^{2g\sqrt{\frac{k_y}{W}}t} - 1}{e^{2g\sqrt{\frac{k_y}{W}}t} + 1}\right\} = \sqrt{\frac{W}{k_y}}\frac{e^{g\sqrt{\frac{k_y}{W}}t} - e^{-g\sqrt{\frac{k_y}{W}}}}{e^{g\sqrt{\frac{k_y}{W}}t} + e^{-g\sqrt{\frac{k_y}{W}}}} = \sqrt{\frac{W}{k_y}}\tanh g\sqrt{\frac{k_y}{W}}t \qquad (7)$$

Writing v = dy/dt and integrating,

$$y = \sqrt{\frac{W}{k_y}}\int \tanh\left(g\sqrt{\frac{k_y}{W}}t\right)dt + A = \frac{W}{kg}\log\cosh g\sqrt{\frac{k_y}{W}}t \qquad (8)$$

The constant of integration being zero if y = 0 at t = 0.

A shorter method is to use the integral form

$$\int \frac{dv}{W - k_y v^2} = \frac{1}{\sqrt{Wk_y}}\tanh^{-1}\left(\sqrt{\frac{k_y}{W}}v\right) = \frac{g}{W}t \qquad (9)$$

The constant of integration being zero since v = 0 at t = 0. Hence

$$v = \sqrt{\frac{W}{k_y}}\tanh\left(g\sqrt{\frac{k_y}{W}}t\right) \qquad (10)$$

Finding v in term of y can be achieved by either eliminating t from the expressions for v and y above, or by writing $\frac{d^2y}{dx^2} = v\frac{dv}{dt}$ in the equation of motion and integrate with respect to y. Each method leads to the equation

$$v^2 = \frac{W}{k_y}\left(1 - e^{-\frac{2gk_y y}{W}}\right) \qquad (11)$$

2.3 UPWARDS SHOOTING OF BODIES

A body of weight W is shot upwards under gravity at a velocity u and air resistance kv^2 where k is a constant. The velocity v is then given by [(1)]

$$v^2 = \left(u^2 + \frac{W}{k_y}\right)e^{-\frac{2k_y g}{W}y} - \frac{W}{k_y}$$

(12)

Whence

$$y = \frac{W}{2k_y g}\log\frac{u^2 + \frac{W}{k_y}}{v^2 + \frac{W}{k_y}}$$

(13)

At apex $v = 0$, hence the maximum height Ymax reached by the body is

$$Y\max = \frac{W}{2k_y g}\log\left(1 + \frac{k_y u^2}{W}\right) \tag{14}$$

A more detailed shorter method is to use the integral form as follows

Let air resistance be $k_y v^2$ when the velocity is v. Note that air resistance and the weight W are both positive acting downwards,

$$-\frac{W}{g}\frac{d^2y}{dx^2} = W + k_y v^2 \tag{15}$$

Writing $$\frac{d^2y}{dt^2} = \frac{dv}{dt} \tag{16}$$

$$-\int\frac{dv}{W + k_y v^2} = \int\frac{g}{W}dt + C \tag{17}$$

Hence $$-\frac{1}{\sqrt{k_y W}}\tan^{-1}\left(v\sqrt{\frac{k_y}{W}}\right) = \frac{g}{W}t + C \tag{18}$$

Since v = u at t = 0, hence

$$C = -\frac{1}{\sqrt{k_y W}}\tan^{-1}\left(u\sqrt{\frac{k_y}{W}}\right) \tag{19}$$

Whence

$$v = \frac{dy}{dt} = \sqrt{\frac{W}{k_y}}\tan\left\{\tan^{-1}\left(u\sqrt{\frac{k_y}{W}}\right) - g\sqrt{\frac{k_y}{W}}t\right\} \tag{20}$$

At apex $v = 0$ and $t = t_{max}$

Therefore

$$t_{max} = \sqrt{\frac{W}{k_y g^2}}\tan^{-1}\left(u\sqrt{\frac{k_y}{W}}\right) \tag{21}$$

Integrating

$$y = \frac{W}{k_y g} \log \cos\left\{ \tan^{-1}\left(u\sqrt{\frac{k_y}{W}} \right) - g\sqrt{\frac{k_y}{W}} t \right\} + B \tag{22}$$

At t = 0 y = 0 then

$$B = -\frac{W}{k_y g} \log \cos\left\{ \tan^{-1}\left(u\sqrt{\frac{k_y}{W}} \right) \right\} \tag{23}$$

Hence

$$y = \frac{W}{k_y g} \log \frac{\cos\left\{ \tan^{-1}\left(u\sqrt{\frac{k_y}{W}} \right) - g\sqrt{\frac{k_y}{W}} t \right\}}{\cos\left\{ \tan^{-1}\left(u\sqrt{\frac{k_y}{W}} \right) \right\}} \tag{24}$$

2.4 MOTION OF BODIES UNDER THE ACTION OF AIR RESISTANCE AND ABSENCE OF GRAVITY

A body of weight W moves horizontally at a velocity v_x and moves against a resistance proportional to the square of its velocity. Its velocity v_x and displacement X at time t are determined as follows [1]

Let the resistance be $k_x v_x^2$ when the velocity is v_x. The equation of motion is

$$\frac{W}{g} \frac{d^2 x}{dx^2} = -k_x v_x{}^2 \tag{25}$$

Writing $$\frac{d^2 x}{dt^2} = \frac{dv_x}{dt} \tag{26}$$

$$-\frac{dv_x}{v_x{}^2} = \frac{k_x g}{W} dt \tag{27}$$

Integrating,

$$\frac{1}{v_x} = \frac{k_x g}{W} t + B \tag{28}$$

Since at t = 0 v_x = Ux then

$$B = \frac{1}{Ux} \tag{29}$$

Hence

$$\frac{dx}{dt} = v_x = \frac{1}{\frac{1}{Ux} + \frac{k_x g}{W} t} \qquad (30)$$

Integrating

$$X = \int \frac{dt}{\frac{1}{Ux} + \frac{k_x g}{W} t} = \frac{W}{k_x g} \log\left(\frac{k_x g}{W} t + \frac{1}{Ux} \right) + F \qquad (31)$$

Since at t = 0 X = 0 then

$$F = -\frac{W}{k_x g} \log\left(\frac{1}{Ux} \right) \qquad (32)$$

Therefore

$$X = \frac{W}{k_x g} \log\left(\frac{k_x g}{W} t + \frac{1}{Ux} \right) - \frac{W}{k_x g} \log\left(\frac{1}{Ux} \right) = \frac{W}{k_x g} \log\left(\frac{k_x g}{W} Uxt + 1 \right) \qquad (33)$$

Note that if the body is a projectile, t is the sum of the times of ascent and descent, and X is its horizontal range.

Let the ordinate of the target be $\pm y$ (+ if y is measured below the origin) then the height of fall Yf is

$$Yf = Y\max \pm y \qquad (34)$$

The time of fall t_f is obtained from equation (8) as

$$t_f = \sqrt{\frac{W}{k_y g^2}} \cosh^{-1} e^{\frac{k_y g}{W} Yf} \qquad (35)$$

Therefore, the total flight time t in equation (29) is

$$t = t_f + t_{max} \qquad (36)$$

and the horizontal range X is

$$X = \frac{W}{k_x g} \log\left[\frac{k_x g}{W} Ux \left\{ \sqrt{\frac{W}{k_y g^2}} \cosh^{-1} e^{\frac{k_y g}{W} Yf} + \sqrt{\frac{W}{k_y g^2}} \tan^{-1}\left(u\sqrt{\frac{k_y}{W}} \right) \right\} + 1 \right]$$

or

$$X = \frac{W}{k_x g} \log\left[\frac{k_x}{\sqrt{k_y W}} Ux \left\{ \cosh^{-1} e^{\frac{k_y g}{W} Yf} + \tan^{-1}\left(u\sqrt{\frac{k_y}{W}} \right) \right\} + 1 \right] \qquad (37)$$

2.5 DRAG CONSTANTS k AND kx

Drag constants are defined as follows

$$k_y = \frac{\rho}{g} A_y C_{dy} \tag{38}$$

$$k_x = \frac{\rho}{g} A_x C_{dx} \tag{39}$$

Where, ρ is the unit weight of air at ambient conditions that may be determined from the following thermodynamic relation [2]

$$\rho = \frac{P}{RT} \tag{40}$$

Since $\rho = 1.293\ \mathrm{kg/m^3}$ at an atmospheric pressure of 760 mm Hg and at an absolute thermodynamic temperature of $273°$ Kelvin[2] ,the gas constant R can then be determined as

$$R = \frac{1(\text{atmosphere})}{1.293\left(\frac{\text{kg}}{\text{m}^3}\right) * 273°(\text{Kelven})} \tag{41}$$

P is the atmospheric pressure measured in atmospheres, T is the absolute thermodynamic ambient temperature in ° Kelvin. A_x and A_y are the projectile maximum cross-sectional areas projected on planes perpendicular to the x - and to the y - axes respectively. C_{dx} and C_{dy} are the dimensionless coefficients of drag in the directions of x - and y - axes respectively. C_{dx} and C_{dy} depend on the shape of the projectile and are normally given by the manufacturer, or may be determined by tests.

CHAPTER 3 HORIZONTAL ANGLE CORRECTIONS

3.1 HORIZONTAL ANGLE CORRECTION DUE TO EARTH'S ROTATION

Distances on great circle may be determined by using the following formula [3]

$$D = \cos^{-1}\left[\sin(LATs)\sin(LATd) + \cos(LATs)\cos(LATd)\cos(LNGd - LNGs)\right] * 60 \qquad \text{...........} (42)$$

Where

D is the distance in nautical miles.
(Note that a nautical mile is the length of the arc of 1 minute of the meridian = 6080 ft = 1.85318 kilometres at the equator) [4]

LATs and LNGs are the latitude and longitude of the source in degrees and fraction thereof.

LATd and LNGd are the latitude and longitude of the destination in degrees and fraction thereof.

Hence, the meridian length at the equator = 1.85318*60*360 = 40,028.688 kilometres
Since the Earth completes a revolution in 24 hours, the speed of a point at the equator is = 40028688/(24*3600) = 463.295 m/s. The speed of rotation Vp of any other point P on the globe, defined by its latitude and longitude is

$$Vp = (1-ABS(LATp)/90)*463.295 \qquad (43)$$

Whence, the horizontal angle correction Ac due to Earth's rotation of a projectile flying a time t is calculated as follows

$$\theta = \sin^{-1}\left\{\frac{(LNGd - LNGs)\left(1 - \frac{LATd}{90}\right) * 60 * 1853.18}{D}\right\} \qquad (44)$$

Where θ is the angle between the x - y plane and the great circle passing through the source. Therefore,

$$Ac = \tan^{-1}\left\{\frac{(Vd - Vs)t\cos(\theta)}{(Vd - Vs)t\sin(\theta) + D}\right\} \qquad (45)$$

Note that if Ac > 0 then the angle correction Ac must be set to the East of the target hitting point. It shall be set to the West of the target hitting point if Ac < 0.

3.2 HORIZONTAL ANGLE CORRECTION DUE TO WIND ACTION

Let the wind velocity be Wv, its incidence be Wi and the time of flight be t. Head wind is defined in this context to have zero incidence. The sign of incidence is positive if measured anti clock wise from the x - axis, and negative if measured clock wise. The horizontal angle correction Hc, is determined from the following formula

$$Hc = \tan^{-1}\left\{\frac{Wv\sin(Wi)t}{X}\right\} \tag{46}$$

The influence of wind Wd, on the range is

$$Wd = -Wv\cos(Wi)t \tag{47}$$

CHAPTER 4 COMPUTER PROGRAM

4.1 INTRODUCTION

A computer program has been developed to solve the projectile equations of motion. Constant gravity and air resistance proportional to the square of the projectile velocity are considered in the solution. The objective was to predict vertical and horizontal shooting angles for accurate artillery aiming at moving and stationary targets.

To satisfy the objectives of the program, efficient iterative routines that reduced computing time to a fraction of a second and the computing errors to less than 10^{-10} were developed.

The program language is HTBasic [5], compatible with Hewlett-Packard [6] BASIC 6.2. The program can run on either Hewlett-Packard workstations or on any IBM PC or 100% compatible. The program is made of a short main program and of two large subprograms. The subprograms can be compiled in machine code and stored in two numeric arrays, thus increasing the performance speed by many folds and providing adequate degree of security.

Two categories of shooting ranges are considered, a short range where horizontal angle corrections due to Earth's rotation are ignored, and long range where such corrections are considered.

4.2 SHORT RANGE

4.2.1 Data input

Two data input forms are displayed on the screen in secession. A flickering cursor takes its proper position in the forms' columns demanding the input of the indicated data. After entering the data into the computer memory, the computer checks the input, rejects all illogical inputs and displays appropriate messages thereat. If the input data is logical it will be printed in colours in its respective column and the cursor moves automatically to the next column. After the completion of filling the data input form, the computer gives the user the following options

To make corrections.
To print the data input forms.
To continue otherwise.

The entered data comprise information about the target status, the atmospheric conditions and the projectile specifications.

4.2.2 Target status

Information about the target status comprises the following data

a) Horizontal distance of the target hitting point from the shooting source.
b) Relative level of the target hitting point with respect to the shooting source.
c) Velocity and incidence of the moving targets.

4.2.3 Atmospheric conditions

Information about atmospheric conditions comprises the following data

a) Velocity and incidence of wind.
b) Average trajectory temperature and pressure.

4.2.4 Projectile specifications

Projectile specifications comprise the following data

a) Muzzle velocity.
b) Dimensionless drag coefficients in the vertical and horizontal directions.

4.3 LONG RANGE

4.3.1 Data input

Two additional data input forms containing the latitudes, longitudes and altitudes above sea level of the source and target hitting point, are added to the data input forms described in Section 2 above. Data is entered into the computer memory in identical manner as described in Section 2. The range and relative level of the target hitting point are not entered from the key board. They are determined by the program from the latitudes, longitudes and altitudes of the source and the target hitting point, and printed in their respective columns automatically.

The units of measurement of latitudes and longitudes are degrees, minutes and seconds. The units of measurement of the altitudes are meters.

4.4 PROGRAM OUTPUT

4.4.1 Fig 1

The performance of the selected gun under the specified conditions is plotted in this figure. The maximum range and the vertical angle thereof and the specified ranges are superimposed thereon. The vertical dotted line representing the specified range may cut the gun’s performance curve in one point or in two points, thus defining the vertical shooting angles. If it does not cut the performance curve, then the selected projectile is deemed inadequate to reach the target hitting point. Continuing program execution under such circumstances, causes the program to halt, and a message to appear on the screen advising the gun’s maximum range under the specified conditions and that the projectile cannot reach its target.

The user has the options of dumping the graphics of this figure to the connected printer or continues program execution to Table 1.

4.4.2 Table 1 (low angle)

If there are low and high vertical shooting angles, the aiming summary relevant to the low shooting angle appear in this Table.

Options of printing this Table and or continuing program execution to Fig 2 (low angle) are available to the user.

4.4.3 Fig 2 (low angle)

Fig 2 plots the trajectory using anisotropic scale. To verify a suspected obstacle in the trajectory, enter its abscissa and ordinate from the keyboard. A vertical solid line representing the obstacle will then be plotted to exact size and location. If the obstacle interferes with the trajectory, or if the user wishes to investigate shooting with the high vertical angle, then press [ENTER]. This moves program execution to the display of Table 1(high angle). If however, there is one shooting angle, then only Table 1 (high angle) will be displayed.

The user has the option of dumping the graphics of this figure to the connected printer or continues to Table 1 (high angle).

4.4.4 Table 1 (high angle)

This Table presents the aiming summary relevant to the high vertical shooting angle.

Options of printing this Table and or continuing program execution to Fig 2 (high angle) are available to the user.

4.4.5 Fig 2 (high angle)

Fig 2 (high angle) plots the trajectory using non-isotropic scale. Verification of suspected obstacles is conducted in the same way as in Fig 2 (low angle). If the obstacle line still interferes with the trajectory, or if the user is not satisfied, then press [ENTER]. This causes the program to halt, and a message advising the user to use a bigger gun to appear on the screen.

If letter y is entered and the target was previously defined a moving target, then Fig 3 and Table 2 will be displayed in graphics. Note that for stationary targets Fig 3 and Table 2 will not be displayed.

The user has the options of dumping Fig 3 and Table 2 to the connected printer, and or ending the session

4.4.6 Fig 3 and Table 2

A graphical representation of shooting moving target is plotted to scale in Fig 3. A summary of shooting moving target is listed in graphics in Table 2.
Fig 3 and Table 2 are presented on the same page.

Options of dumping the content of the screen to the connected printer and or ending the session are available to the user.

CHAPTER 5 EXAMPLES

All data displayed in the Data Input Forms 1 to 4, are rounded for convenience to three decimals. However, for maximum accuracy no rounding of data takes place in the program computations.

5.1 SHORT RANGE

Target status	Atmospheric conditions	Projectile specifications
Abscissa 3000 m	Wind velocity 20 m/s	Area projection
Ordinate 100 m	Wind incidence -12°	in y - z plane 0.1 m^2
Velocity 300 m/s	Trajectory temperature 0°	in x - z plane 0.2 m^2
Incidence 45°	Trajectory pressure 1 Atm	Drag coefficient
		in x - axis direction 0.05
		in y - axis direction 0.08
		Projectile weight 20 kg
		Muzzle velocity 1000 m/s

A suspected mountain summit located at 2400 m from the gun's muzzle and 102 m above the gun's muzzle is suspected to be an obstacle in the trajectory.

This moving target is considered suitable for short range aiming. The above listed information was entered in Data Input Forms 3 and 4.

As shown in Fig 1, the target may be shot in either low or high vertical angle. Table 1 (low angle) gives the shooting output summary for a stationary target. Fig 2 represents the trajectory. The co-ordinates of the suspected obstacle were entered into the computer memory. The program superimposed on Fig 2 (low angle) a vertical line representing the suspected obstacle. In this particular case, the obstacle interferes with the trajectory, hence the high vertical angle was selected as a solution.
Table 1 (high angle) and Fig 2 (high angle) are then printed. Fig 3 represents the geometry of shooting the moving target and Table 2 gives the final shooting summary of the moving target.

5.2 LONG RANGE

Source/Destination	Atmospheric conditions	Projectile specifications
Source	Wind velocity 20 m/s	Area projection
Latitude 15°20'13.1"	Wind incidence -12°	in y - z plane 0.1 m^2
Longitude 37° 14'0"	Trajectory temperature 0°	in x - z plane 0.2 m^2
Altitude 0 m	Trajectory pressure 1 Atm	Drag coefficient
Destination		in x - axis direction .01
Latitude 15°0'0"		in y - axis direction .02
Longitude 37° 0'0"		Projectile weight 100 kg
Altitude 0 m		Muzzle velocity 1000
m/s		

This target is assumed to be a suitable long range stationary target. Data Input Forms 1, 2, 3 and 4 are then duly filled with the information above listed. No obstacle is suspected to interfere with the trajectory.

It is established from Fig 1 that the target may be shot in either low or high vertical angle. Since no obstacle is suspected, the low shooting angle was selected. Table 1 gives the shooting output summary for a stationary target and Fig 2 represents the projectile trajectory.

REFERENCES

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2 - Staff of Research and Education Association.”Handbook of Mathematical, Scientific, and Engineering Formulas, Tables, Functions, Graphs, Transforms”. Research and Education Association, 61 Ethel Road West, Piscataway, New Jersey 08854, USA, 1988.

3) Hewlett-Packard Owner’s Handbook and Programming Guide, HP-41C, March 1980.

4) Rolls-Royce Conversion Booklet, Rolls Royce Ltd., Derby.

5) HTBasic, DOS 386/486 Version for IBM PC compatible computers. TranseEra Corporation, 345 E 800 S, Orem, Utah 84058-9914, USA, 1994.

6) HP BASIC 6.2 for HP9000 workstations. Hewlett-Packard Company, 3404 East Harmony Road, Fort Collins, Colorado 80525, USA, 1990

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SHORT RANGE SHOOTING

DATA INPUT FORM 3. ARTILLERY AIMING AT MOVING/STATIONARY TARGETS

Target				Average ambient atmospheric properties			
co-ordinates		Velocity	Incid-ence	Wind velocity	Incid-ence	Bullet path temp	Bullet path pressure
Abscissa m	Ordinate m	m/s	degrees	m/s	degrees	°C	atmosphere
3000	100	300	45	20	-12	0	1

DATA INPUT FORM 4. PROJECTILE SPECIFICATION

Area projection		Dimensionless drag coefficients		Weight	Muzzle velocity
on y - z plane m^2	on x - z plane m^2	in x - axis direct.	in y - axis direct.	kg	m/s
.1	.2	.05	.08	20	1000

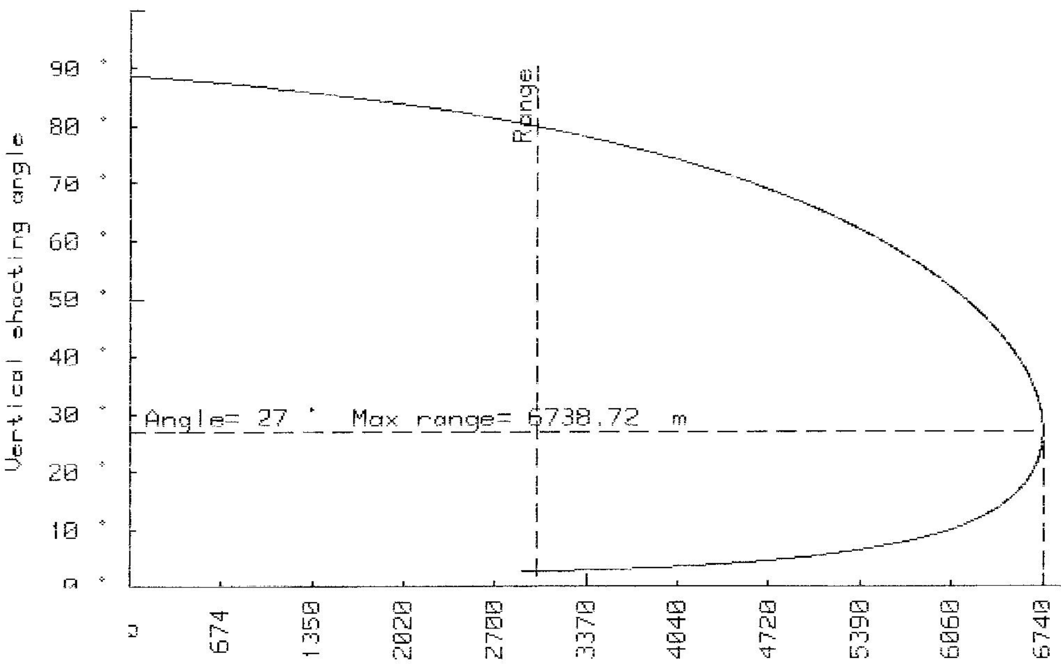

Fig 1 Projectile performance diagram

Table 1 Shooting fixed target output summary (low angle)

Target input specification		
Specified range	3000	m
Relative level	100	m
Velocity	300	m/s
Incidence	45 °	
Output data		
Drag constants		
in vertical direction: ky=Γ/g*Ay*Cdy	.00210887	kg-s^2/m^2
in horizontal direction:Kx=Γ/g*Ax*Cdx	.000659021	kg-s^2/m^2
Apex height above firing source	103.463	m
Time to reach apex	4.51128	s
Time of fall from apex to target	.840706	s
Total flight time	5.35199	s
Computed range	3000	m
Max range under the specified criteria	6738.72	m
Vertical angle at max range	27 ° 0 ′ 4.63E-6 ″	
Vertical shooting angle at target	2 ° 43 ′ 37.1 ″	
Horizontal angle to resist wind offset	-1 ° 34 ′ 29.9 ″	
Impact horizontal velocity component	366.144	m/s
Impact vertical velocity component	8.22767	m/s

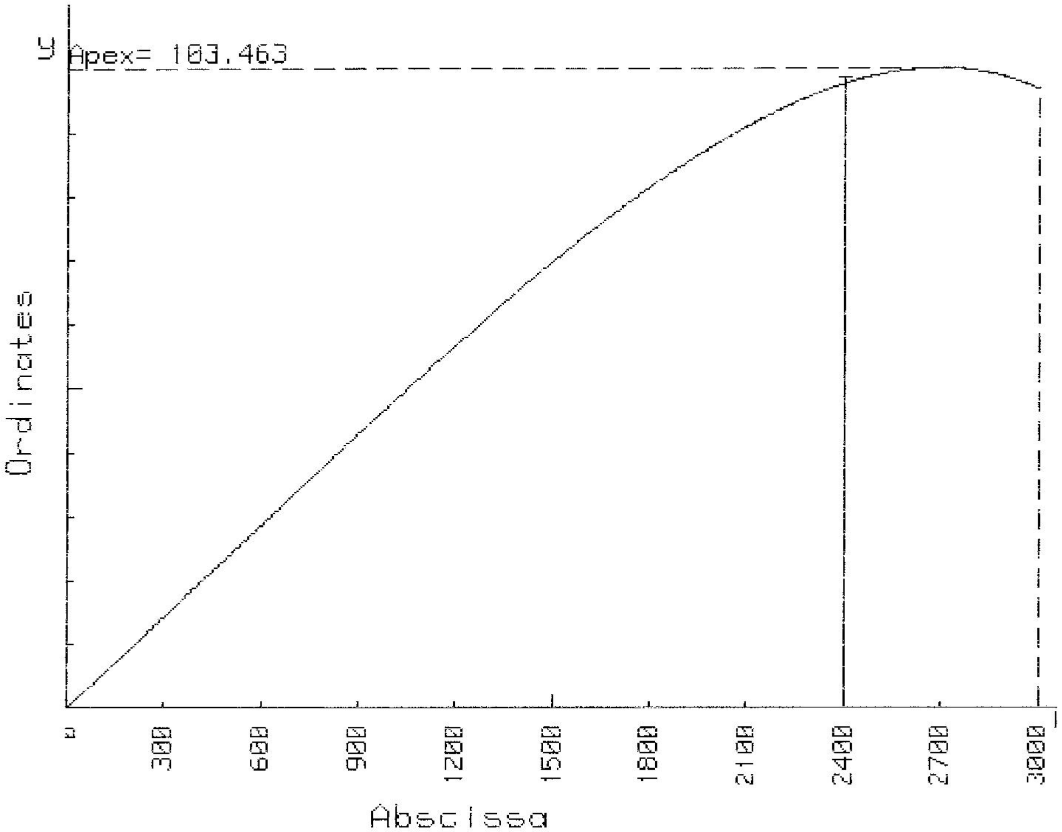

Fig 2 Missile path diagram (low angle)

Table 1 Shooting fixed target output summary (high angle)

Target input specification	
Specified range	3000 m
Relative level	100 m
Velocity	300 m/s
Incidence	45 °
Output data	
Drag constants	
in vertical direction: $ky=\Gamma/g*Ay*Cdy$	.00210887 kg-s^2/m^2
in horizontal direction: $Kx=\Gamma/g*Ax*Cdx$	.000659021 kg-s^2/m^2
Apex height above firing source	2241.14 m
Time to reach apex	14.6145 s
Time of fall from apex to target	28.8376 s
Total flight time	43.4522 s
Computed range	3000 m
Max range under the specified criteria	6738.72 m
Vertical angle at max range	27 ° 0 ' 4.63E-6 ''
Vertical shooting angle at target	2 ° 43 ' 37.1 ''
Horizontal angle to resist wind offset	-4 ° 33 ' 12.1 ''
Impact horizontal velocity component	50.6855 m/s
Impact vertical velocity component	96.8025 m/s

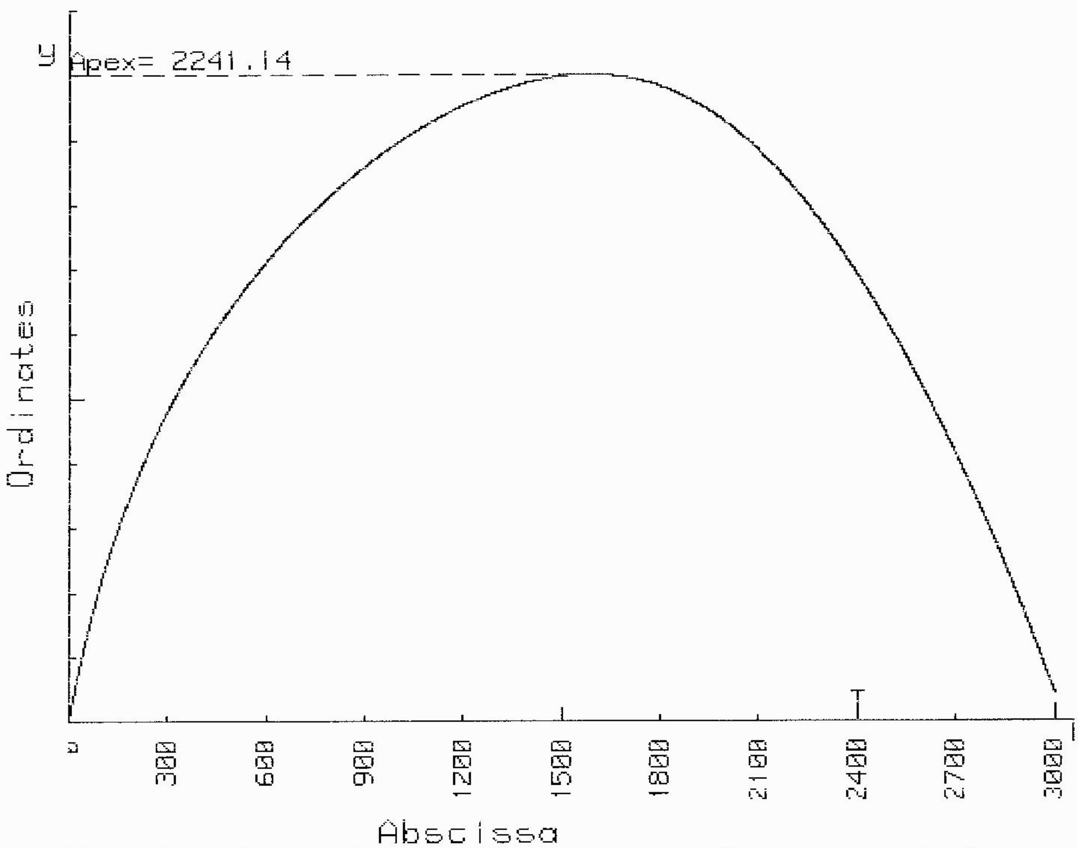

Fig 2 Missile path diagram (high angle)

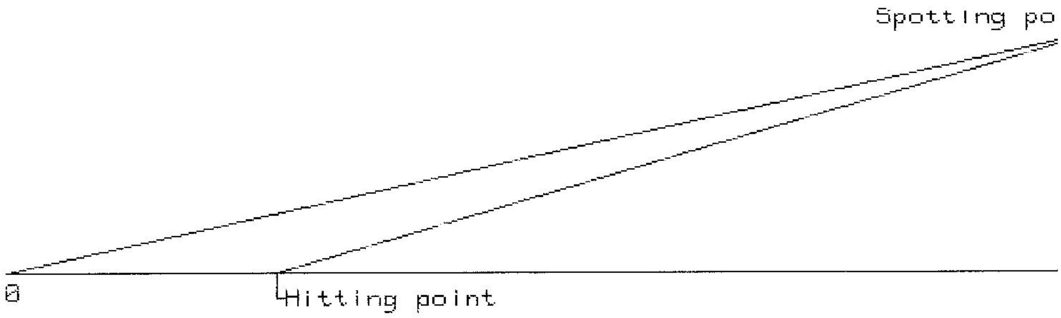

Fig 3 Moving target shooting diagram

Table 2 Shooting moving target summary

Target velocity	300 m/s
Target incidence	45 ° 0 ′ 0 ″
Vertical shooting angle at target	2 ° 43 ′ 37.1 ″
Horizontal angle to resist wind effect	-4 ° 33 ′ 12.1 ″
Spotting point angle	37 ° 1 ′ 50.1 ″
Final horizontal angle setting	33 ° 35 ′ 10.2 ″
Spotting point distance from origin	15304.7 m
Hitting point distance from origin	3000 m
Distance between spotting and hitting	13035.6 m

LONG RANGE SHOOTING

DATA INPUT FORM 1. LONG RANGE ARTILLERY

Source						
Latitude			Longitude			Altitude above sea m
degrees	minutes	seconds	degrees	minutes	seconds	
15	20	13.1	37	14	0	0

DATA INPUT FORM 2. LONG RANGE ARTILLERY

Destination						
Latitude			Longitude			Altitude above sea m
degrees	minutes	seconds	degrees	minutes	seconds	
15	0	0	37	0	0	0

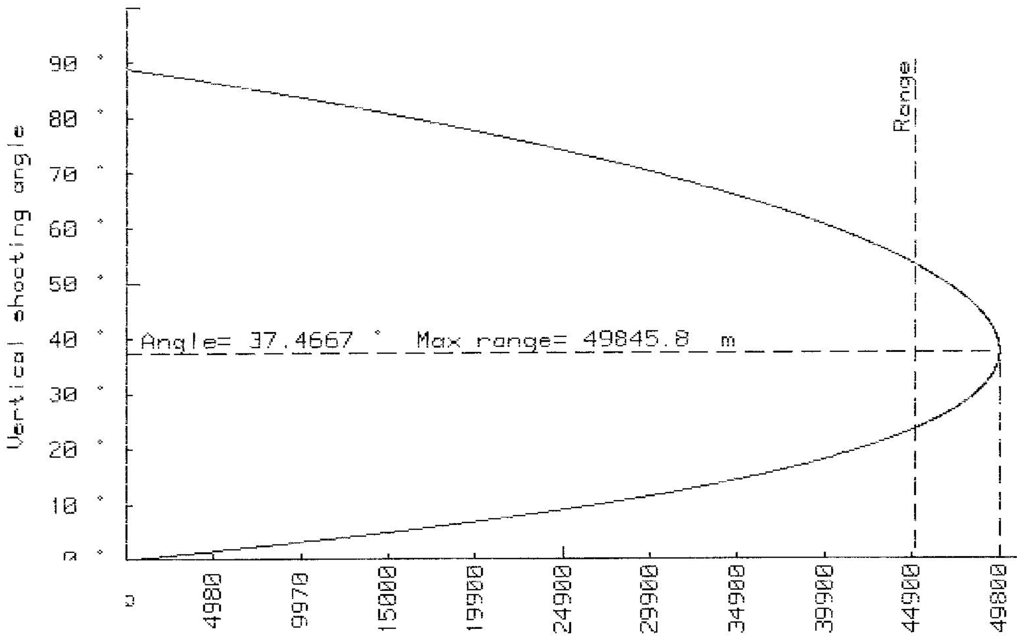

Fig 1 Projectile performance diagram

Table 1 Shooting fixed target output summary (low angle)

Target input specification	
Specified range	45065.481 m
Relative level	0 m
Velocity	0 m/s
Incidence	0 °
Output data	
Drag constants	
in vertical direction: ky=Γ/g*Ay*Cdy	.000527217 kg-s^2/m^2
in horizontal direction:Kx=Γ/g*Ax*Cdx	.000131804 kg-s^2/m^2
Apex height above firing source	5894.04 m
Time to reach apex	32.9332 s
Time of fall from apex to target	36.4497 s
Total flight time	69.3829 s
Computed range	45065.481 m
Max range under the specified criteria	49845.8 m
Vertical angle at max range	37 ° 28 ' 4.63E-6 ''
Vertical shooting angle at target	23 ° 31 ' 23.5 ''
Horizontal angle to resist wind offset	-1 ° 37 ' 59.5 ''
Hor angle setting due to Earth rotation	0 ° 0 ' 9.61 ''East
Impact horizontal velocity component	503.08 m/s
Impact vertical velocity component	294.248 m/s

DATA INPUT FORM 3. ARTILLERY AIMING AT MOVING/STATIONARY TARGETS

Target				Average ambient atmospheric properties			
co-ordinates		Velocity	Incid-ence	Wind velocity	Incid-ence	Bullet path temp	Bullet path pressure
Abscissa m	Ordinate m	m/s	degrees	m/s	degrees	°C	atmosphere
45100	0	0	0	20	-12	0	1

DATA INPUT FORM 4. PROJECTILE SPECIFICATION

Area projection		Dimensionless drag coefficients		Weight	Muzzle velocity
on y - z plane m^2	on x - z plane m^2	in x - axis direct.	in y - axis direct.	kg	m/s
.1	.2	.01	.02	100	1000

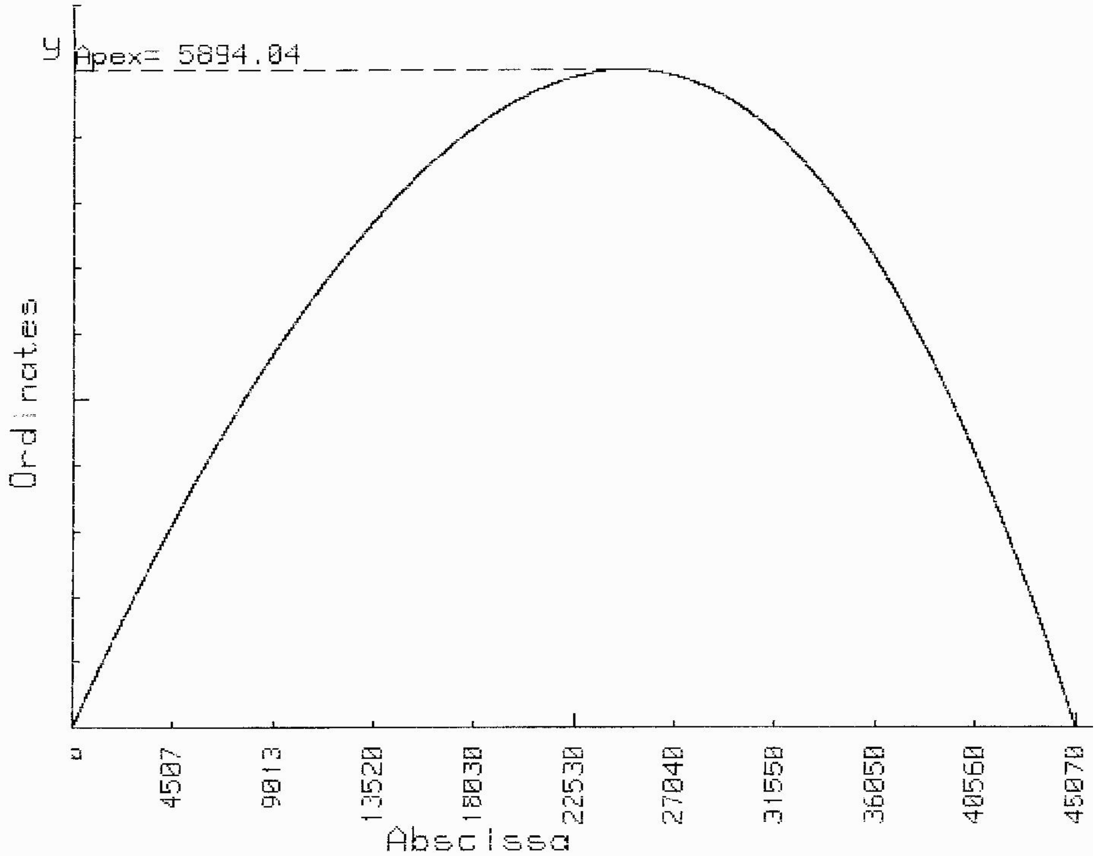

Fig 2 Missile path diagram (low angle)