THE MATHEMATICAL CONCEPT OF BLACK HOLES

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NOMENCLATURE

g Gravity acceleration given to a unit mass by unit force = 32.174 ft/sec^2
k gravitational constant, (in cgs $6.664x10^{-8}$)
r Radius of black hole
F component of force of attraction in the direction of motion
m mass of moving particle
v *ds/dt* = velocity of moving particle.
s distance from a fixed point
V velocity of escape
R Earth's radius

CONVERTING THE EARTH INTO A BLACK HOLE

1 INTRODUCTION

Defining a Black Hole as a star whose mass collapsed and became extremely dense (say 10^{27}) and its gravity became so great that no light can escape its field. In this study an attempt shall be made to determine the hypothetical size of the Earth when it collapses into a Black Hole.

2 NEWTON'S LAW OF MOTION [1]

Newton's second law of motion states that the time rate of change of momentum is equal to the impressed force. In differential form

$$\frac{d}{dt}(mv) = F \tag{1}$$

where,

F = component of force of attraction in the direction of motion
m = mass of moving particle
$v = ds/dt$ = velocity of moving particle.

It is supposed that the particle moves in a straight line, its distance from some fixed point on that straight line is s.

3 NEWTON'S LAW OF GRAVITY [1]

The law states that two bodies attract each other with a force proportional to the product of their masses m_1 and m_2 and inversely proportional to the square of the distance between them. If the force of attraction is denoted by F, and the distance between them by s then,

$$F = k\frac{m_1 m_2}{s^2} \tag{2}$$

where, k is called a gravitational constant, ($ft^4/lb - sec^4$)

Substituting for F in equation (1) gives

$$\frac{d}{dt}(mv) = k\frac{m_1 m_2}{s^2} \tag{3}$$

Differentiating equation (3) gives

$$v\frac{dm}{dt} + m\frac{dv}{dt} = k\frac{m_1 m_2}{s^2} \tag{4}$$

Since the masses m_1 and m_2 are constants, therefore $dm/dt = 0$, and equation (4) may then be re-written as

$$m\frac{dv}{dt} = k\frac{m_1 m_2}{s^2} \tag{5}$$

Focussing our attention on Earth and particles falling on it, then the mass of Earth m_1 is much larger than the mass of the falling particle m_2. Therefore, the acceleration dv/dt is towards the centre of Earth and it corresponds only to m_2. The acceleration of Earth toward the particle may be ignored. Hence

$$\frac{dv}{dt} = k\frac{m_1}{s^2} \tag{6}$$

or, $km_1 = s^2 dv/dt$.

At the Earth's surface the acceleration of gravity $dv/dt = g$, and, s = the Earth's radius, R. Therefore,

$$km_1 = gR^2 \tag{7}$$

Ignoring air resistance and all other forces and assuming that the forces acting on the falling particle are only gravitational, then substitute in Equation (6) gives

$$\frac{dv}{dt} = \frac{gR^2}{s^2} \tag{8}$$

Equation (8) gives the acceleration of the particle at any distance s from the Earth's centre.

3.1 Velocity of escape

The acceleration dv/dt may be re-written as follows

$$\frac{dv}{dt} = \frac{ds}{dt}\frac{dv}{ds} = v\frac{dv}{ds} \tag{9}$$

Substituting in equation (8) and separating the variables, we get in integral form

$$\frac{1}{gR^2}\int_0^V v dv = \int_r^\infty \frac{ds}{s^2} \tag{10}$$

Integrating equation (10) gives

$$\frac{V^2}{2gR^2} = \frac{1}{r} - \frac{1}{\infty} \tag{11}$$

where,

$\frac{1}{\infty} = 0$

V = the Earth's escape velocity

$r = R$ at the Earth's surface

The velocity of escape V, then becomes

$$V = \sqrt{2gR} \tag{12}$$

Taking the Earth's radius as 3977 miles and the acceleration of gravity as 32.2/5280 miles/sec^2 then the velocity of escape from the Earth is approximately:

$$V = \sqrt{2 * 32.2 / 5280 * 3977} = 6.96 miles / \sec \tag{13}$$

4 RADIUS OF BLACK HOLE EARTH

According to the theory of Black Holes, converting the Earth into a black hole would keep its mass unchanged. Only its diameter becomes much smaller due to the collapse of its matter. By definition, the velocity of escape becomes at least equal to the velocity of light so that no light can escape its field of gravity. Hence, black holes will only grow in size by devouring matters from other heavenly objects.

Equation (11) may be re-written as

$$r = \frac{2gR^2}{V^2} \tag{14}$$

where,

r = the radius of Earth after converting it into a Black Hole

V = the velocity of light, 984,251,968 ft/sec

g = the acceleration of gravity, 32.2 ft/sec^2

R = Earth's radius, 3977 miles = 20998560 ft

Substituting in equation (14), gives

r = 0.03 ft = 9 mm approximately (15)

5 FLAWS IN THE BLACK HOLE THEORY

Each atom is made up of a combination of electrons, which are very light and have negative charge, protons, which are much larger than electrons and have positive charge, and neutrons, which have no charge but are as large and as heavy as protons.

The process of converting an atom into a black hole atom is assumed to be achieved by removing the electrons from the atom structure and compressing the protons and neutrons to form the dense matter of black hole. Since the mass of electrons is negligible in comparison with the mass of protons and neutrons, the mass of the black hole atom shall be assumed to be equal to the mass of the atom before

conversion. However, the forces of attraction at the surface of the black hole atom are assumed to become so great that they attract other atoms and grow in size.

5.1 ENERGY OF IMPACT [1]

The theory of formation of black holes does not take into account the energy required to cause the collapse of matter into a black hole. Since no external source of energy is used in this process only converting the matter of the atoms into energy would be assumed.

Differential equation (1), $F = \frac{d}{dt}(mv)$ may be rewritten as

$$F = \frac{d}{dt}(mv) = \frac{d(mv)}{ds}\frac{ds}{dt} \tag{16}$$

Substituting ds/dt by v then multiplying both sides of equation (16) by ds, equation (16) becomes

$$Fds = vd(mv) = v(vdm + mdv) \tag{17}$$

Since the mass of the atom is constant then dm = 0, hence equation (17) becomes

$$Fds = mvdv \tag{18}$$

Integrating differential equation (18) gives

$$\int_{S_0}^{S} Fds = m\int_{V_0}^{C} vdv = \frac{1}{2}mC^2 - \frac{1}{2}mV_0^{\,2} \tag{19}$$

Equation (19) represents the law of conservation of energy. The right side is the change in kinetic energy, while the left side represents the work done when the atom moves from S_0 to S. Thus the left side is the change in potential energy. It is therefore evident from above that equation (19) is reversible if F(s) is continuous.

Assuming the atom to be at rest before being attracted by the black hole atom, then the quantity $\frac{1}{2}mV_0^{\,2} = 0$

It should be noted that the forces of attraction acting on the atom and those acting on the black hole atom are equal in magnitude but opposite in direction. Since the masses of the atom and the black hole atom are equal then the change in the kinetic energies of both atoms would be equal. Therefore, the amount of total energy, E released at impact is

$$E = 2\left(\frac{1}{2}mC^2\right) = mC^2 \tag{20}$$

E is indeed equal to the energy released when all the mass of the attracted atom is converted into energy. Therefore, the mass of this atom would disappear after impact and the assumed Black Hole atom can never gain any additional matter.

Had the Lord's gracious Hands converted atoms into black hole atoms, their radius would have been approximately equal to $\frac{0.03}{3977*5280} = 1.43*10^{-9}$ of the atom original radius. Such black hole atom will not be able to grow bigger than its size and therefore would have no significance in the Universe.

6 CONCLUSION

Since matter and energy are in fact different forms of the same thing. Matter can be turned into energy, and energy into matter. The energy released at impact, would be equal to the total kinetic energy of the moving masses. Hence, the matter of the particle would be turned into energy and no black hole could possibly be formed. The mass would disappear and become energy after impact.

REFERENCES

1 Sokolnikoff, I. S. and Redheffer, R. M.: Nathematics of Physics and Modern Engineering, McGraw-Hill, 1958.